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## The Evolution of Knowledge: A Scientific Meeting in Honor of Jürgen Renn

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# The Evolution of Knowledge:
# A Scientific Meeting in Honor of Jürgen Renn

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# The Tangled Roots of the New Mathematics of the 17th Century Scientific Revolution


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In the *Discours préliminaire* to the *Encyclopédie* (*Encyclopédie* I: xxvi), Jean le Rond d'Alembert writes as follows:[2]

> Finally Newton arrived, for whom Huygens had prepared the road, giving to philosophy a shape that it seems she will keep. This great genius saw it was time to ban from physics conjectures and vague hypotheses, or at least not take them for more than they were worth, and that this science should be submitted to nothing but the experiences of geometry.

We notice how an echo of Nicolas Boileau's *Enfin Malherbe vint*, "finally Malherbe arrived" (ed. Chéron 1861, 93) gives extra weight to the praise of Newton as the definitive culmination of the scientific revolution (d'Alembert knew his *belles-lettres* just as well as his mathematics). A naive reading might further find a specific reference to Newton's use of *geometric* proofs in the *Principia*, in contrast to the application of infinitesimal calculus, but the defining contrast with conjectures and vague hypotheses shows that this is overly naive. Infinitesimal analysis was perfectly at home in the *Classe de Géométrie* of the Académie des sciences, to which d'Alembert belonged, as it was in d'Alembert's own writings. D'Alembert does to Newton what Sainte-Beuve (Chéron 1861: ix) claims Boileau has done to Malherbe's prescription: *il l'étend et l'approprie à son siècle*, "he extends him and takes possession of him for his own century." And in d'Alembert's 18th century that meant that the permanent shape of natural philosophy brought about by Newton was


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[2] My translation, as all translations in the following. The original runs:

> Newton, à qui la route avoit été préparé par Huyghens, parut enfin, & donna à la Philosophie une forme qu'elle semble devoir conserver. Ce grand génie vit qu'il étoit tems de bannir de la Physique les conjectures & les hypothèses vagues, ou du moins de ne les donner que pour ce qu'elles valoient, & que cette Science devoit être uniquement soûmise aux expériences de la Géométrie.

now expressed in the new infinitesimal analysis, whereas the arguments of Galileo, Kepler, and Huygens had indeed been just as geometric as those of Newton.[3]

That the miscellaneous infinitesimal considerations we find in the 17th century gave rise to *infinitesimal analysis* was conditioned by the preceding creation of the new *algebraic analysis.* This, as well as the preceding consideration concerning d'Alembert, I shall leave as a postulate, easily verified however by a glance at Leibniz's mathematical texts. My topic, less worked out by others, is the complex process from which emerged the first,+ algebraic level of the new analysis of the 17th century.

Complex process? Isn't it quite simple? Al-Khwārizmī created algebra in the 820s; Abū Kāmil refined it; Fibonacci reordered it in Latin in 1202 (or 1228); it survived with little change and no progress for three centuries—and then Cardano (in some nasty interaction with Tartaglia) brought it to a new level, inspiring Viète and Descartes. That is the standard story.[4]

Standard stories are not necessarily wrong, but this one is.

There is not much reason to discuss the Arabic developments in any detail, since algebra was received in Catholic-Christian Europe only through three channels—efficiently through two only.

One was Gerard of Cremona's translation of al-Khwārizmī's algebra, made in Toledo around 1170. It did not circulate much—there was not really space for it within the world of university learning, but it did circulate modestly; 15 manuscripts survive (Hughes 1986: 221).

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[3] *More* geometric, we may say: according to Guicciardini (2016), it seems that Newton was right when claiming in later times to have possessed the fluxion technique already when producing the *Principia* in 1687.

[4] The second half of the story underlies this passage from (Karpinski 1929):

> From the mathematical point of view this treatise by Jacob of Florence, like the similar arithmetic of Calandri, marks little advance on the arithmetic and algebra of Leonard of Pisa. The work indicates the type of problems which continued current in Italy during the thirteenth to the fifteenth and even sixteenth centuries, stimulating abler students of mathematics than this Jacob to researches which bore fruit in the sixteenth century in the achievements of Scipione del Ferro, Ferrari, Tartaglia, Cardan and Bombelli.

If even Louis Karpinski knew no better, who in his time would have known better?

Another translation was prepared by Robert of Chester (ed. Hughes 1989); we know it from three 15th-century manuscripts produced in southern Germany; its particular terminology has left no traces whatsoever. The algebraic problems contained in the *Liber mahameleth* (ed. Sesiano 2014) and the few pages introducing the topic in the second part of the *Liber algorismi* (ed. Burnett, Zhao, and Lampe 2007, 163–65) were equally ineffectual.[5]

In 1202, Fibonacci wrote a first version of the *Liber abbaci*, in the last chapter of which an algebra is contained. A revised version was made in the late 1220s. The best guess is that the basic introduction was produced independently by Fibonacci under inspiration from Gerard's translation;[6] the illustrating problems were borrowed from many sources, some of them indirectly from Abū Kāmil (Høyrup 2022b). A large cluster of problems borrowed together, probably inserted in the revised edition, was based on a Latin translation of an Arabic treatise drawing upon Abū Kāmil while revising his approach; neither the Latin translation nor the Arabic original are known to have survived. Fibonacci's algebra, on its part, did survive as part of the larger treatise; its impact, however, was negligible—Jean de Murs drew on it as one of several sources for the algebraic books of his (scarcely influential) *Quadripartitum numerorum* in the mid-14th century (ed. L'Huillier 1990), and it was partially copied by Benedetto da Firenze and a few others in the mid-15th century, yet without affecting their own algebraic work.

The essential reception was effectuated by a handful of abbacus masters in the early 14th century.

[5] Probably during the second quarter of the 13th century, Guglielmo de Lunis made another translation of al-Khwārizmī's algebra into either Latin or (rather) the vernacular (Høyrup 2022a, 313–317). Guglielmo *may* have drawn upon Gerard's translation, but certainly also had direct acquaintance with Arabic material. Longer or shorter stretches from its beginning were quoted by Benedetto and two more around 1460, but it has left no other traces. A redaction of al-Khwārizmī's work known as *Liber restauracionis* (Moyon 2019) is probably also to be dated to the 13th or 14th century (possibly, as pointed out by Moyon, a Latin translation of an Arabic redaction). It awoke enough interest to be translated into the vernacular around 1400—but since its particular notation did not spread further, the interest seems to have subsided soon afterwards.

[6] This seems to follow from Nobuo Miura's analysis (1981).

The immediate source area (a single source can be excluded) must have been Romance-speaking[7] and located somewhere in the Ibero-Provençal area. Compared with Abū Kāmil, the level is modest.

Geometric proofs are absent. Rules are given for the six basic first- and second-degree "cases" (equation types) and for those cases of the third and fourth degree that can be reduced to these or solved by means of a root extraction. Soon, however, (false) rules were also offered for such higher-degree cases that cannot be resolved in these ways. At the conditions of the time, they were not easily controlled: the proposed solutions all contain radicals, and radicals were never approximated (and higher-degree equations were never used for any practical purpose). Such false rules might therefore be useful in competitions for positions in municipal abbacus schools and for students.

Very soon we also see incipient use of abbreviations for the algebraic powers, used in particular in formal calculations—for example[8] the reduction:

$$\frac{360}{1\rho}+\frac{360}{1\rho\ m\hat{e}\ 6}=\frac{1080\ m\hat{e}\ 2160}{2\ censi\ m\hat{e}\ 6\rho}$$

(ρ stands for the thing; *mê* for *meno*, "less"; + and = are modern; the fraction lines are in the original). Since very similar notations had been created in the Maghreb in the outgoing 12th century (too late for Fibonacci to know them), it seems almost certain that the ideas were borrowed.

However that may be, some abbacus writers used abbreviations in a way that effectively barred their use for symbolic calculations; nobody used them systematically (Høyrup 2010; 2015). Nor was there any agreement about what the abbreviations should be. This was still the situation in 1494, when Luca Pacioli (1494: 67$^{v}$) summed up the situation in the words *tot capita tot sensus*, "as many heads, so many opinions."

Beyond using the abbreviations in formal calculations (which, by the way, could also be performed with the names written in full, as Biagio does with *censi*), in the outgoing 14th

[7] There is not a single Arabic loanword in the texts; for the second power of the unknown *cosa*, "thing," they use *censo*, rendering the Latin translation *census* of Arabic *māl* that had been the Toledo standard in the later 12th century.

[8] Siena, Biblioteca Comunale L.IV.21, fol. 404$^{v}$, Benedetto da Firenze rendering what Biagio "il vecchio" had written before ca. 1340.

century we also encounter schemes (emulating those used in the arithmetic of Hindu-Arabic numerals) for the addition and multiplication of polynomials—even they agreeing with Maghreb models. From the later 14th century onward, we know some scattered instances of the use of several unknowns (essential in Viète's and Descartes's algebras), and from Pacioli we know that more must have existed; even this, however, was never systematized—Pacioli just informs us so that we may know, so he says.

The source area that inspired the beginning of abbacus algebra must have understood the nature of the sequence of algebraic powers as a continued proportion—that is shown by the rules for reducible higher-degree cases. This is not strange; this insight had been well described by al-Karajī and had spread from him to Arabic algebra. Also other aspects of early abbacus algebra make one think of a "diluted al-Karajī" (Høyrup 2011). It is far from certain, however, that the first generation of abbacus-algebra writers understood what they were borrowing (if they had, the acceptance of the false rules is hard to explain[9]). In 1344, however, Dardi of Pisa showed in his formulation of rules for a huge number of cases involving roots of powers that he understood to the full, at least practically[10]—but he never explains the principles involved; that had to wait for Antonio de' Mazzinghi's work half a century later.[11]

Although the system shows its first cracks, Antonio's naming of the higher powers is generally multiplicative—his "cube of cube" is the sixth, not the ninth power. In other words, the powers *thing*, *censo,* and *cube* are entities, not functions.[12] That was to change over the next century, but once again not systematically and not in all writers. Pacioli has

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[9] For instance, understanding would reveal that the problems:

$\alpha C = \beta t+N$ and $\alpha K = \beta t+N$

($t$ being the unknown *thing*, $C$ its second and $K$ its third power) can only be solved according to the same rule if $\alpha C = \alpha K$, that is,

- if $\alpha = 0$ (which would be meaningless at the time and is in any case excluded since $\beta$, $t$, and $N$ are all presupposed to be positive)
- or if $C = K$, that is, if $t = 0$ (still excluded by the number concept of the time) or $t = 1$; that is, all in all, if $\alpha = \beta+N$.

[10] (Van Egmond 1983) contains an overview.

[11] Antonio's explicit understanding may have links to his production of the first tables of composite interest.

[12] Some writers also give pseudo-multiplicative names to higher roots, speaking, e.g., of the fifth root as "root of cube root"; others are aware that root-taking is *an operation* and roots therefore by necessity functions (evidently not using this much later term). Antonio introduces the term *radice relata* for the fifth root, and in parallel (that is the crack just mentioned) speaks about the fifth power as the *cubo relato*. Otherwise, his naming for powers remains multiplicative.

come to see the powers as functions, which evidently entails the question of how to name the fifth and higher prime powers. An alternative system explained by Pacioli therefore identifies the powers with their number in the sequence—*number* being the first, which means that Pacioli's number-names are not exponents, and that the easy rule for multiplying by adding exponents does not apply.

Some writers, understanding that the false rules *were* false, tried to find better ways. One method consisted in transforming homogeneous equations—for instance, taking in a problem about a capital growing over 3 years from £100 to £200, not the value after one year but the interest per month as unknown; in mathematical principle this is a linear transformation, and the one who did it must have had a very good understanding of polynomial algebra (*very good* indeed since the transformation had to be done without symbols).[13] Whether the inventor understood that the resulting rule was not generally valid is not clear, but Dardi (from whom we know about these rules) knew.

Another way to advance consisted in the invention of specious "roots"—the "cube root of 44 with added 5" being 4 because $4^3 = 44+5\cdot4$. In itself this is just a name for the solution to the case "cube equal to roots and number," but at least one treatise from around 1400[14] shows that it can also be used to solve the case "cube and *censi* equal to number"—even this is achieved by a linear substitution and thus asks for mastery of polynomial algebra.

The texts do not explain the methods—only the non-reduced coefficients reveal to us how the special rules and the case-transformations were obtained. In consequence these ingenious methods apparently did not spread in the environment, and Cardano had to reinvent. There was absolutely no impact on the German *coß*, the next phase in the story.

The *coß* descended from abbacus algebra but in an intricate and protracted process. Beginning around 1450, a number of German mathematical writers—Friedrich Amann, Johannes Regiomontanus, and several anonyms (Amann and Regiomontanus at least with background in university culture and astronomy)—took interest in algebra,

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[13] Analysis in (Høyrup 2022a, 224–226).

[14] Florence, BNC, fondo princ. II.V.152. (Franci & Pancanti 1988) contains an edition of the extensive algebra section of the manuscript. Analysis of this aspect of the algebra in (Høyrup 2022a: 246*f*).

apparently as a new mathematical discipline they wanted to learn about. The first decades of the reception mirror the messy state of abbacus algebra. The terminology not only reflects inspiration from northern Italy (whose "thing" was *cossa*) as well as Florence (where it was *cosa*); both Amann and Regiomontanus would use several different sets of abbreviations, evidently corresponding to their source of the moment; some of the anonyms are more messy. There is no reason to be scandalized: what these pioneers drew on was equally confused (none of them seem to have had the good luck to stumble upon a high-level abbacus algebra); before the Germans could produce coherence of their own, they had to make sense of whatever they had been able to find.

Eclecticism did not last many decades, however. In 1489 Johannes Widmann, university educated, published the first large-scale *Rechenbuch*. It contains no algebra, but already in 1486 Widmann had held algebra lectures at Leipzig university. We do not have any text showing what he taught except for the announcement referring to "the 24 rules of algebra, and that which they presuppose"—the latter specified to include "algorism for fractions, ratios and surds." He can be supposed to have built on a manuscript in his possession that still survives—a manuscript which taken as a whole is quite eclectic. But we may further suppose that his lectures were in the style of his book, and thus systematic (as also suggested by the announcement). Widmann probably used the standard notation for powers that we know from manuscripts dated from the following years (+ and – were in any case used in his *Rechenbuch*).

University lectures were held in Latin. A Latin algebra from no later than 1504 was almost certainly written by Andreas Alexander (Folkerts 1996), one of the first specialized mathematics lecturers in Leipzig. A related German text about the topic (the *Initius algebras*[15]) may also be from his hand—if not, somebody else profited from Alexander's work (as actually suggested by some of the formulations).

Both of these works reduce the number of rules to 8, taking advantage of reducibility through division. Neither circulated much; the latter at least was used by Adam Ries, who produced the oldest surviving manuscript copy and probably used both for his *Coss;*[16]

[15] (Ed. Curtze 1902, 435–609).
[16] Thus spelled (in agreement with the title page) so as to distinguish it from the general discipline *coß*.

even that work, however, did not circulate, so the only lasting influence of Alexander's work was the inspiration Christoph Rudolff received from it. We may observe, however, that Alexander may have learned from the higher level of Italian abbacus algebra, which he seems to have digested though with some approximation.[17]

Rudolff, beyond the "8 rules," took over and established the standard notation for the algebraic powers definitively; these notations were still standard when a disgusted Descartes had to learn them in the Jesuit school. Rudolff also borrowed the schemes for polynomial arithmetic familiar in Italy at least since 1400.

However, before saying more about Rudolff's discipline-defining work we should notice Heinrich Schreyber's *Ayn new kunstlich Buech, welches gar gewiß und behend lernet nach der gemainen regel Detre, welschen Practic, regel falsi unn etlichen regeln Cosse* from 1518 (published under Schreyber's Latinized name Grammateus in 1521). This is a general *Rechenbuch*, but written before norms crystallized as to what such a book should contain; beyond an extensive algebra, it also contains Boethian music theory and bookkeeping, otherwise strangers to the *Rechenbücher*. As Alexander's algebra, that of Schreyber describes the arithmetic of polynomials by means of schemes, and offers a restricted set of rules (seven only). Noteworthy, however, is the notation for the algebraic powers: instead of the already established standard symbols, Schreyber uses abbreviated ordinal numbers, corresponding to exponents: *N*, *pri*, *2a*, *3a*, *4a*, etc. It is obvious from the texts that both Alexander and Schreyber came from a university background; the latter studied at Vienne University from 1507 onward and taught there from 1517 to 1521 (Kaunzner 1970, 229; Vogel 1975, 589).

Rudolff was taught by Schreyber, as he tells (1525: 204); whether he frequented the university directly is unclear, we know almost nothing about his biography. There is no doubt, however, that he knew basic university mathematics—algorism as well as the Boethian naming of ratios, both of which he extends: the former (as "algorism of the *coß*) he uses as a framework also for the arithmetic of monomials and polynomials; the latter he refers to also when considering ratios between broken numbers.

[17] That is, if we disregard the "historical" beginning of the *Initius algebras*, which is charmingly hilarious and quite different from anything Italian I know.

Beyond such extensions, Rudolff creates no new mathematical knowledge, but he provides order and structure. Beyond what was already said, he teaches the use of a second unknown (actually more than two unknowns, but used in a way where never more than two are in play at a time, so two names suffice—Pacioli [1494, 191$^{v}$] had done the same); while predecessors had simply done so without taking much notice, Rudolff states that this technique is "a completion of the *coß*, indeed in truth a completion without which it would not be worth much more than a trifle [*pfifferling*]."

Rudolff's book became and remained the defining basis for the *coß*. Schreyber's book was reprinted several times, but nobody took over his symbolism; in the rudimentary presentation of algebra in the *Deutsche Arithmetica* from 1545 Stifel suggested to use the names *sum*, *sum.sum* and *sum.sum.sum* instead of *radix*, *census,* and *cubus*, with no more effect. In 1553, when Rudolff's book, long of print and not to be found "even at triple or quadruple price" (an indication of its status), Stifel produced an "improved much augmented" new edition of the cherished work, from which this quotation is taken (fol. A 2$^{v}$).

Before that (namely in [1544]), Stifel had published the *Arithmetica integra*. Stifel there acknowledges the importance of Rudolff's *Coss*, but he goes far beyond it—for instance by dealing in depth with *Elements* X transformed into arithmetical theory. He further invents a letter-based notation for many variables that allows higher powers and products, without using it himself for anything spectacular.[18]

In his expositions of algebra from 1550 and 1551—the former printed in Basel, the latter in Paris—Scheubel does not advance on Rudolff and Stifel within algebra proper. His integration of algebra into *Elements* I–VI in the former is restricted to the insertion of numerical examples, going beyond advanced current practical geometries (and Heron's *Metrica*, unknown at the time) only by including radicals and binomials in the range of accepted numbers. The separately republished algebraic introduction from 1551 (reprinted in Paris in 1552, evidence that the book sold well) exhibits Humanist aspirations, firstly (fol. 2$^{r}$) by endorsing Regiomontanus's ascription of algebra to

[18] It had little immediate impact but may have inspired Jean Borrel (Buteo 1559), who like Stifel makes use of capital letters. Borrel's notation is borrowed with due reference by Guillaume Gosselin (1577, 80$^{r}$), and Gosselin *may* again have provided inspiration for Viète's letter symbolism.

Diophantos, secondly by including a number of Greek arithmetical diagrams provided with Latin translations and algebraic solutions.

Jacques Peletier tells us in *L'Algebre* from 1554 what was accessible in France. Peletier knows Pacioli's *Summa* and Cardano's *Ars magna*, and from Cardano he knows about Fibonacci. He also knows Stifel and Scheubel, and he has heard about Rudolff, Ries, and Nuñez but not seen their books (at the time, that of Nuñez was indeed an unpublished manuscript). Peletier himself takes Stifel as his basis, using also his symbolism and, even as classicizing condiments (fols $24^{r}$, $76^{r}$), an arithmetical riddle about Alexander the Great and the philosopher Calisthenes and the story about Archimedes and Hieron's crown—the latter going back to Rudolff.[19]

To judge from the technical terminology, Viète's primary reference for algebra was Gosselin, who knows Stifel, Cardano, and Peletier.[20] Descartes learned algebra in his Jesuit school, La Flèche, from Christophorus Clavius's *Algebra*. Even this is in the general style of Rudolff and Stifel. Clavius, a great pedagogue, makes his own formulations, but for instance his way to deal with negative numbers and negative powers (Clavius 1608, 28–29) leaves no doubt that he had the *Arithmetica integra* (here [Stifel 1544, $249^{r-v}$]) on his desk while writing.

So, what Viète (1591, $2^{v}$) experienced as "an old art defiled and befouled by barbarians" and what Descartes (1637, 19) described as "a confused and obscure art that puts the mind in difficulty instead of a science that cultivates it" was not Arabic algebra but *abbacus algebra transformed and put into order* as *coß*, and to some extent as unfolded by Cardano.

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[19] (Stifel 1544, $234^{r}$, $267^{r}$) and (Rudolff 1525, $84^{r}$). Peletier knows that the story comes from Vitruvius's *De architectura* IX, while Rudolff, even more precise, states that "I have read in Vitruvius, in the third chapter of the ninth book of his *Architecture*." Stifel has nothing. It appears that Peletier has looked up the details in Vitruvius's text.

[20] Thus, (Gosselin 1577, $47^{v}$). Gosselin has also read (Nuñez 1567; thus fol. $67^{r}$) but that is not a main inspiration. Gosselin's terminology for several unknowns comes from Borrel, as said in note 18; for a single unknown it *might* have been taken from (Ramus 1560), not least the term *latus* for the first power (for which Borrel used a Florentine ρ, while his second power is ♦). In principle, Viète's *latus* might thus come from Gosselin as well as Petrus Ramus, but there is so little substance in Ramus's primer (and only one unknown) that Viète could have learned nothing even if he should happen to have known this anonymous piece.

This putting into order was effectuated by writers like Alexander, Schreyber, Rudolff, Stifel, and Cardano, not university teachers all of them but all strongly influenced by the Boethian-Euclidean norms of the university tradition. What Viète, Descartes, and their ilk knew as interesting mathematics was already different, however—we might speak of it as "Humanist mathematics," rather perhaps as "post-Humanist."

Humanism had always been centered on the "civically useful" as understood in courtly culture. Around 1500 it had become clear that Latin letters might perhaps still be "a weapon more to be feared than a troop of horses," as claimed by the Chancellor of Florence in 1406 (Gragg 1927, x)—but Latin letters were definitely no match for the French artillery, nor did they help much when the Portuguese and Spanish courts engaged in transoceanic travel. It was also during the years around 1500 that Pacioli, putting into writing a century's experience of architects and military and hydraulic engineers, reinterpreted Aristotle's opinion that mathematics is the most certain of sciences as a claim that all the other sciences derive from mathematics.[21]

In consequence of such experience, some Humanists or court mathematicians with a Humanist bent engaged in publishing and translating the Greek mathematical classics. Bartolomeo Zamberti's problematic translation of the *Elements* was published in 1505, and Grynaeus's complete edition of the Greek Euclid with Proclos's commentary in 1533. The *editio princeps* of Pappos's *Collection* appeared in Basel in 1538—Commandino's Latin translation in 1588, after having circulated in manuscript. The *editio princeps* of Archimedes was printed in 1544; Memmo's Latin edition of books I–IV of Apollonios's *Conics* appeared in 1537 (that of Commandino in 1566); Xylander's Latin translation of Diophantos was published in 1575 (the Greek *editio princeps* only in 1621). When Viète reached mathematical maturity, a rather full range of the Greek mathematical classics was thus within his reach.

However, this acquisition of new material was relevant for the transformation of algebra only because the mathematical undertaking itself had changed. The medieval university taught theory in lectures, and disputations and their written emulation in *quaestiones*

[21] (Pacioli 1509, $2^r$) in the dedicatory letter to Ludovico Sforza—written in 1498, before Sforza lost his court and Pacioli his position as a court mathematician precisely to the French artillery.

invited metamathematical reflection about the status of the object and objects of the theory. That is also what we see in Scheubel's volumes: the algebra does not change the geometric theory, nor does the Humanist orientation expressed in the inclusion of Greek epigrams affect the algebra.

The metamorphosis of the mathematical undertaking is epitomized in the famous concluding line of Viète's *Isagoge* (1591, 9$^r$): *nullum non problema solvere*, "to leave no problem unsolved." The mathematics of Viète, his antagonist Adriaan van Roomen, and later Descartes, Fermat, etc. was centered on *problems* within an agonistic culture which made van Roomen the *antagonist* of Viète.[22]

The Italian abbacus culture, too, had been agonistic—the abbacus masters challenged each other with higher-degree algebraic problems and difficult versions of "the purchase of a horse," "the finding of a purse," and other recreational classics. This led to the invention of specious roots and made Benedetto da Firenze create a notation for first-degree algebra with up to five unknowns. But nobody appears ever to have noticed Benedetto's innovation—one reason at least being the exiguous number of practitioners who were at a level where they might have understood and appreciated it. The culture of the German *Rechenmeister* probably had a higher density; since it was a print culture, at least it had much more efficient communication. It was not intellectually agonistic, however; books were competing on the book market and were therefore almost invariably marketed on their title page as *new*. In this environment, the specious roots had no social role, and they never left their abbacus home.

The culture of Viète and his kind was agonistic once again; but now its problems were those inspired by the Greek geometers. *That* was what inspired Viète and Descartes to create their very different versions of the new algebra, which turned out to be the hoped-for tool for problem solving within the new mathematics, just as abbacus and *Rechenmeister* algebra had been an efficient tool for solving traditional problems.

[22] The details about the confrontation between van Roomen and Viète add an extra twist (Busard 1975, 533; 1976, 22). It was brought about by an ambassador from the Netherlands who boasted to Henri IV that France did not possess a geometer able to solve a problem suggested by Adriaan van Roomen. That is, mathematical prowess was now taking over the symbolic power of Latin letters. I shall not pursue the question *why* this agonistic culture arose; still, this episode illustrates that it was not just a mathematicians' fashion.

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